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The Circle-Apollon's Theorem, Equanimous Inverses

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Abstract

Soon, the edition will have the challenge of publishing the prompts of the authors. The theorem of Thales and the theorem of Pythagoras do not escape the rule, and they seem to have been preceded by several millennia. Asking the true paternity of those geometrical realities makes it possible to show that $a*b=R^2$ according to the terms defined by the Theorem of the Circle or Theorem of Apollo stating the constant product of two segments of perpendicular tangents for a given circle. The rigorous demonstration, especially its reciprocal that allows a new mathematical way of drawing the circle, will be mastered by the Artificial Intelligence of Analytical Geometry. We show its discovery as well as the oddity of the equanimous inverses which require the Golden Ratio as the first of them.

Subject Areas

Geometry, Golden Ratio, History

Keywords

Pythagoras-Thales Theorem, Computational Cartesian Analytic Geometry, IM 67118, Plimpton 322, Pythagoras Reality, Pythagoras Theorem, Thales Reality, Thales Theorem, Snefru Papyrus, Trigonometry, Emmanuel's Theorem, Apollon's Theorem, Shiva's Theorem, Equanimous Inverses, Golden Ratio, Reefering, Quadrature du Cercle, Chord Quest, New Way to Draw the Circle, Les 2 Brins de Tangentes Perpendiculaires, Administrative Theorem

1. Introduction

One of the 2 proofs, you are a real teacher at work. Certainly, the theorems of

Thales and Pythagoras are 2 transcendental truths that precede and surpass them even if they are at the origin of the commonly accepted proof sufficiently reasoning to elect them millennia to the rank of theorem. As sure as America was discovered by the Vikings after the Amerindians, the lack of purpose in questioning the paternity of these two transcendental truths didn't allow for finding their fruit with the Circle during all the enlightened centuries. The Artificial Intelligence of Analytical Geometry thus summons truths sufficiently complex so that it is possible to call these basic bricks Axioms even if this gives them back their previous millenary status. The current state of archaeology and Egyptology allows us to attempt this appellation of the Axiom of Ur and Axiom of Snefru even if this order of rights presided over the discovery of Apollo's Theorem. The theorem is stated in part 4, $a*b=R^2$, and we leave to Analytical Geometry its best proof.

We finally consider how the Golden Ratio is the first of bizarre functions that preserve the decimal part or Equanimous Inverses.

2. Pythagoras Reality

The Pythagoras Reality allows drawing the circle according to the Cartesian orthogonal formula $x^2+y^2=R^2$ what draws $y=\pm\sqrt{R^2-x^2}$ [1].

2.1. IM 67118 and Plimpton 322

IM 67118, also known as Db_2-146, is an Old Babylonian clay tablet in the collection of the Iraq Museum that contains the solution to a problem in plane geometry concerning a rectangle with a given area and diagonal. In the last part of the text, the solution is proved correct using the Pythagorean theorem.

The most renowned of all mathematical cuneiform tablets since it was published in 1945, Plimpton 322 reveals that the Babylonians discovered a method of finding Pythagorean triples, that is, sets of three whole numbers such that the square of one of them is the sum of the squares of the other two [2] [3].

2.2. Actual Proof of Pythagoras Reality

A proof that comes with **Figure 1** and should have been the one of Pythagoras, is as follows:

The area of the square c^2 is equal to the area of the largest square minus the areas of the 4 triangles.

The area of the biggest square:

$$(a+b)^2=a^2+b^2+2*a*b$$

The surface of the 4 triangles:

$$4*\frac{a*b}{2}=2*a*b$$

So

$$c^2=a^2+b^2 \tag{1}$$

if, and only if, the triangle is right.

We will call this property "Axiom of Ur". The confidence in Pythagoras {psychedelic}. Also according to its phonetic significance in French.

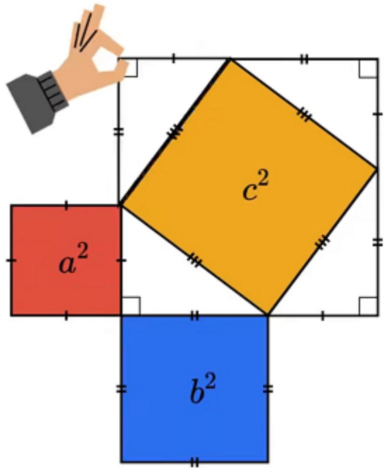

Figure 1. The schematic diagram of pythagoras theorem[1].

3. Thales Reality

If two parallel lines intersect two secant lines, then the ratio of the lengths of the segments on one secant line is equal to the ratio of the lengths of the corresponding segments on the other secant line, and the two parallel segments defined by the two secants are also in that proportion.

3.1. Snefru Papyrus

The different parts of the Eye of Horus taken as a character allow to write fractions including the inverses of the unit (See **Figure 2**). It seems that the Rhomboidal Pyramid {Bent Pyramid} shelters this transcendental mathematical reality. While the pyramid of Meidum was probably transformed from a stepped pyramid to a gently sloping pyramid by the artifice of a filling. The Rhomboidal Pyramid, with its 2 slopes, requires for the artifice of cutting these blocks with a gently sloping face, the knowledge of the reality of Thales, applied to the large format plan of the pyramid and its projections for each of its slopes. This artifice of calculating the parallel dimensions/proportions to report the cutting triangle on each gently sloping block appearing on the side, is taken up with a single slope in the Red Pyramid. We call, this reality of Thales, Proportions Axiom or Axiom of Snefru.

It seems to be historical the story of the trip to Egypt of Thales, where he measures the height of the big pyramid by applying The Axiom of Snefru to the triangles with the shadow of a stick. The gently sloping pyramid knows the axiom within itself.

[1]The schematic diagram of Pythagoras Theorem was found in a course's advertisement for http://brilliant.org/.

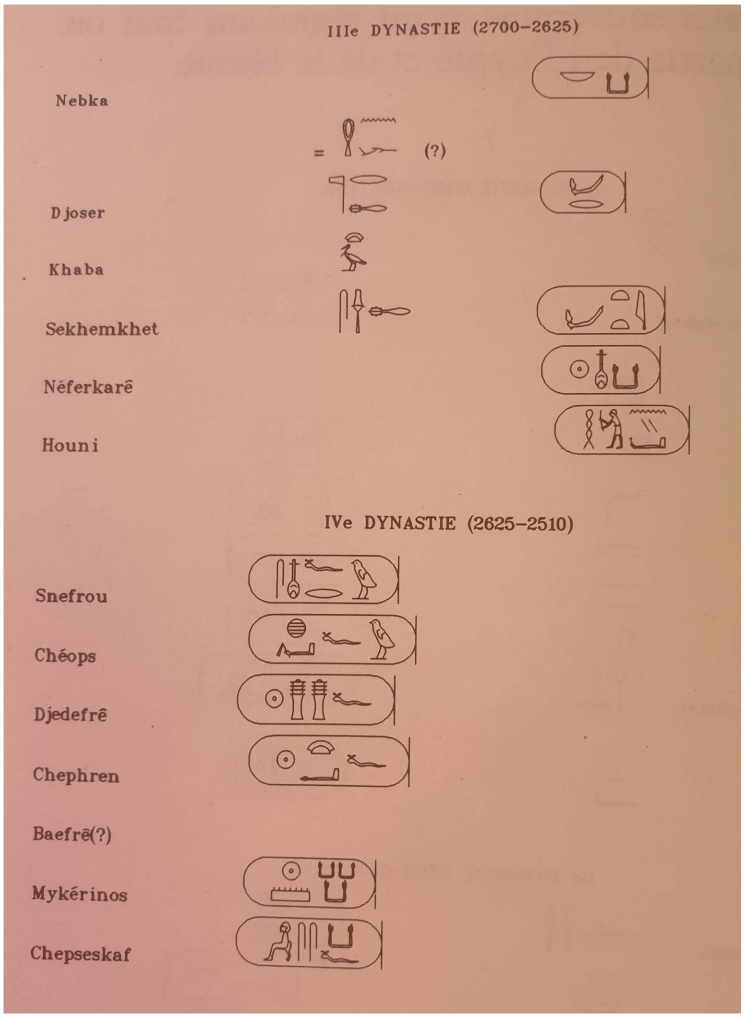

Figure 2. The 3rd and 4th dynasties [4], later it will be the tiger head about.

3.2. Modern Proof of Thales Reality

According to Figure 3, the Thales Theorem states that the following lengths are proportional

$$\frac{AD}{AB}=\frac{AE}{AC}=\frac{DE}{BC} \quad (2)$$

when (DE) is parallel with (BC).

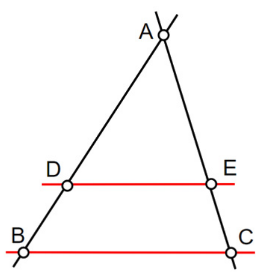

Figure 3. The schematic diagram of Thales Theorem [5].

This is still true when the point A is between the parallel lines (DE) and (BC) with different notations.

It seems that the first proof we have of Thales reality, and maybe allegedly the one of Thales himself, involves invoking "similar triangles", whose angles are the same. However, we might prefer a more formal approach, like the one used in modern teaching by Art of Mathematics on the internet, and maybe more vectorial than what was believed to be from Thales, that is as follows:

$$\mathbf{AD} = \alpha\mathbf{AB} \quad \alpha > 0$$

$$\mathbf{AE} = \beta\mathbf{AC} \quad \beta > 0$$

$$\mathbf{DE} = \gamma\mathbf{BC} \quad \gamma > 0$$

$$\mathbf{DA} + \mathbf{AE} = -\alpha\mathbf{AB} + \beta\mathbf{AC}$$

$$\mathbf{DE} = -\alpha\mathbf{AB} + \beta\mathbf{AC}$$

$$\gamma\mathbf{BC} = -\alpha\mathbf{AB} + \beta\mathbf{AC}$$

$$\alpha\mathbf{BA} + \beta\mathbf{AC} = \gamma\mathbf{BC}$$

$$\text{Let} \quad m = \beta - \alpha$$

$$\alpha\mathbf{BA} + (m + \alpha)\mathbf{AC} = \gamma\mathbf{BC}$$

$$\alpha\mathbf{BA} + \alpha\mathbf{AC} + m\mathbf{AC} = \gamma\mathbf{BC}$$

$$\alpha(\mathbf{BA} + \mathbf{AC}) + m\mathbf{AC} = \gamma\mathbf{BC}$$

$$\alpha\mathbf{BC} + m\mathbf{AC} = \gamma\mathbf{BC}$$

$$\text{Implies} \quad \alpha = \gamma \quad \text{and} \quad m = 0$$

$$m = \beta - \alpha = 0$$

$$\text{Implies} \quad \alpha = \beta = \gamma$$

$$\mathbf{AD} = \alpha\mathbf{AB}$$

$$\mathbf{AE} = \alpha\mathbf{AC}$$

$$\mathbf{DE} = \alpha\mathbf{BC}$$

$$\|\mathbf{AD}\| = \alpha\|\mathbf{AB}\|$$

$$\|\mathbf{AE}\| = \alpha\|\mathbf{AC}\|$$

$$\|\mathbf{DE}\| = \alpha\|\mathbf{BC}\|$$

What implies:

$$AD = \alpha AB$$

$$AE = \alpha AC$$

$$DE = \alpha BC$$

Which implies:

$$\frac{AD}{AB} = \alpha$$

$$\frac{AE}{AC} = \alpha$$

$$\frac{DE}{BC} = \alpha$$

$$\text{So } \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC} \tag{2}$$

3.3. The Scene of Concentric Circles

The exercise with radians shows that if the two parallel lines are replaced by two concentric arcs with the center at the vertex of the triangles, then the proportion of the lengths applies to the lengths of the two arcs. The exercise with radians draws the circle in its own way, with cosine and sine. To establish the use of radians, you have to show that the unit circle has a perimeter of $2\pi R$ by iterating the measurement of the chord c for half the angle each time and stating that the perimeter is equal to n times the nth of the circle approximated by the nth chord. You would need a compass for that.

By the way, an analytical circle is also the set of points in the plane such that the angle to the two endpoints of the diameter is a right angle, on both sides of the diameter.

4. Apollon's Theorem or Emmanuel's Theorem

For given

"a", the straight section tangent to the circle between the extension of the radius of the angle and the perpendicular bisector of the long side of the right triangle inscribed in the circle, so with the diameter for hypotenuse.

And

"b", the straight section tangent to the circle between the extension of the radius of the angle and the perpendicular bisector of the short side of the right triangle inscribed in the circle, so with the diameter for hypotenuse.

Then:

$$a * b = R^2 \tag{3}$$

with R being the radius of the circle.

It's the set up prompt for computational cartesian analytical geometry, that will, by the way, give a correct proof of this Theorem.

We also have for the circle of radius 1 $2a * 2b = 4$.

We must be able to show that $a * b = R^2$ is a necessary and sufficient condition to be a circle.

4.1. Discovery, the Necessary Condition

To prove that the lengths $a * b = R^2$, we firstly take the circle with the generated by the center angle 90°, isosceles right triangle inscribed in it (**Figure 4**).

First, we calculate the hypotenuse of the upper right isosceles triangle by applying the Axiom of Ur to the triangle.

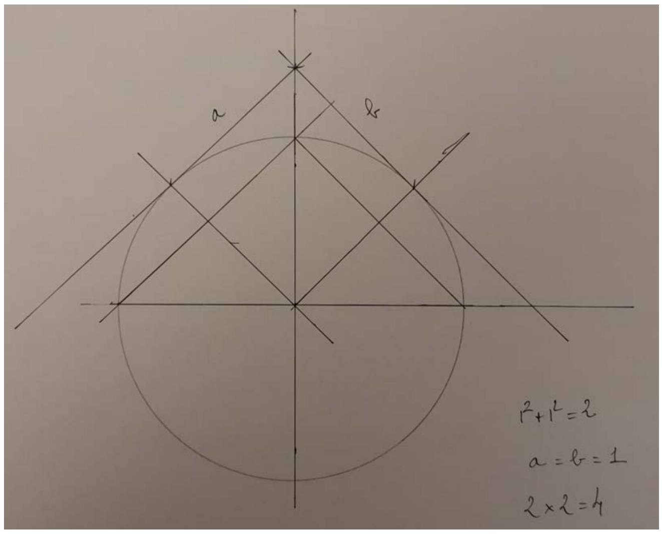

Figure 4. Figure with central angle at 90˚ or $\triangle_4$.

$$R^2 + R^2 = 2R^2 \text{ so the hypotenuse is } R\sqrt{2}.$$

Calculation of the height h on the perpendicular bisector of the hypotenuse of the right isosceles triangle. We apply the Axiom of Ur to the small right triangle:

$$\left(\frac{R\sqrt{2}}{2}\right)^2 + h^2 = R^2$$

$$h^2 = R^2 - \frac{2R^2}{4} = \frac{R^2}{2}$$

$$h = \frac{R}{\sqrt{2}}$$

We have, with the Axiom of Snefru in the canonical triangle:

$$\frac{\frac{R}{\sqrt{2}}}{R} = \frac{\frac{R\sqrt{2}}{2}}{a}$$

So $a = R$

By symmetry of the figure

$$a = b = R$$

$$\text{So } a * b = R^2 \quad (3)$$

We now show that $a * b = R^2$ for **Figure 5** with the central angle at 60˚. We apply the Axiom of Ur to the right triangle inscribed in the circle to find the length of the long side of the right triangle inscribed in the circle, knowing that we know that the short side of this triangle has length R because it is the regular side of the hexagram inscribed in the circle.

$$(2R)^2 - R^2 = 3R^2$$

The length of the long side of the inscribed right triangle is $R\sqrt{3}$.

The length of the perpendicular bisector of this side, from the long side of the triangle in question to the center of the circle, is $\frac{R}{2}$ as the length of the half-side of the hexagram inscribed in the circle.

Applying "Snefru" to canonical right triangles with two parallel sides along *a*, we have

$$\frac{\frac{R}{2}}{R} = \frac{\frac{R\sqrt{3}}{2}}{a}.$$

Thus $a = R\sqrt{3}$.

H is the height of the perpendicular bisector of the small side of the inscribed triangle, it is equal to half the length of the large side of the inscribed triangle

$$H = \frac{R\sqrt{3}}{2}.$$

Applying "Snefru" to canonical right triangles with two parallel sides along *b*, we have

$$\frac{\frac{R\sqrt{3}}{2}}{R} = \frac{\frac{R}{2}}{b}.$$

Thus $b = \frac{R}{\sqrt{3}}$.

So we have well $a * b = R^2$ (3)

In **Figure 5** with the central angle at 60˚, it is shown that the bisector of the central angle intercepts the chord *c*.

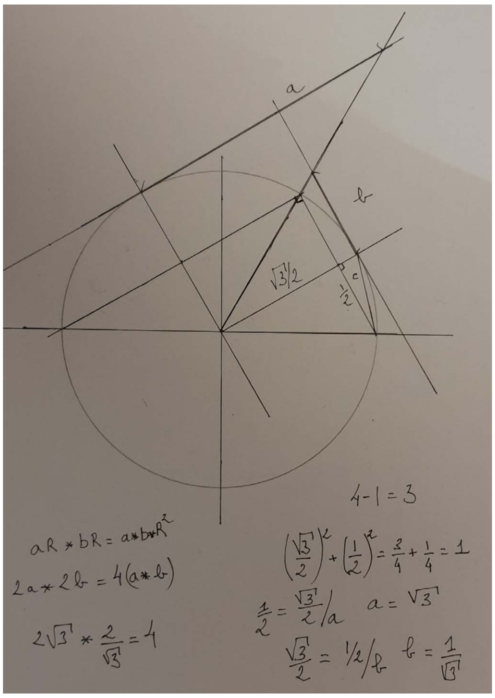

Figure 5. Figure with central angle at 60˚ or $\triangle_6$.

We find c as the chord of the angle 30˚.

By the Axiom of Ur with the triangle whose hypotenuse is c:

$$\left(\frac{R}{2}\right)^2 + \left(R - \frac{R\sqrt{3}}{2}\right)^2 = c^2$$

$$\frac{R^2}{4} + R^2 + \frac{R^2 3}{4} - R^2\sqrt{3} = c^2$$

$$c^2 = 2R^2 - R^2\sqrt{3}$$

$$c^2 = R^2\left(2-\sqrt{3}\right)$$

$$c = R\sqrt{2-\sqrt{3}}\ .$$

We now take **Figure 6**.

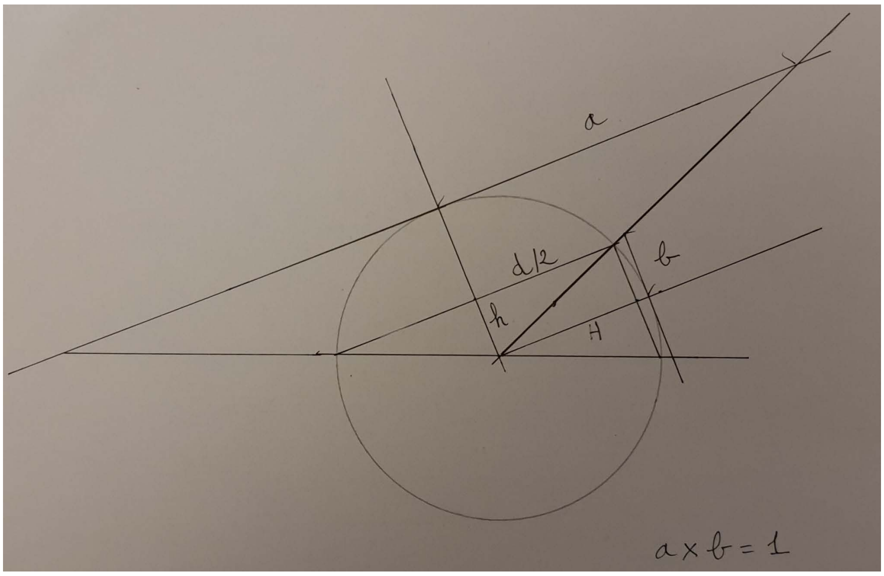

Figure 6. False figure, the reasoning sees an angle at 30˚ when it is 45˚.

d is the long side of the right triangle inscribed in the circle.

By the Axiom of Ur in the triangle inscribed in the circle whose short side is c.

$$d^2 + R^2\left(2-\sqrt{3}\right) = 4R^2$$

$$d^2 = \left(2+\sqrt{3}\right)R^2 .$$

Thus $d = \sqrt{2+\sqrt{3}}R$ and the height $H = \frac{d}{2} = \frac{\sqrt{2+\sqrt{3}}}{2}R$.

We also have $h = \frac{c}{2} = \frac{\sqrt{2-\sqrt{3}}}{2}R$.

By Snefru's Axiom in Canonical Right Triangles, we have:

$$\frac{\frac{d}{2}}{a} = \frac{h}{R} \text{ and } \frac{\frac{c}{2}}{b} = \frac{H}{R} .$$

So $a=\frac{R\frac{d}{2}}{h}=\frac{R^2\frac{\sqrt{2+\sqrt{3}}}{2}}{\frac{\sqrt{2-\sqrt{3}}}{2}R}=\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}R$ and

$$b=\frac{R\frac{c}{2}}{H}=\frac{R^2\frac{\sqrt{2-\sqrt{3}}}{2}}{\frac{\sqrt{2+\sqrt{3}}}{2}R}=\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}R.$$

What hence shows that $a*b=R^2$ (3)

$a*b=R^2$ The circle is surrounded by the external tangents. The figures and calculations above show that one can iterate the calculation for the central angle of half value each time, starting from the first or second figure, and thus reach any angle, however small. This is not worth a demonstration and Analytical Geometry will be better able to do so.

4.2. The Trigonometric Proof

We will use trigonometric functions to prove that $a*b=R^2$ for any given circle. We know cosine and sine are respectively, for one of the other angle of a right triangle:

$$\text{cosine}=\frac{\text{adjacent side}}{\text{hypotenuse}}.$$

And

$$\text{sine}=\frac{\text{opposite side}}{\text{hypotenuse}}.$$

So the little side of the right triangle inscribed in the circle Figure 7 is

$$\text{hypotenuse}*\sin\theta=2R\sin\theta.$$

We apply the Axiom of Ur in the right triangle with the hypotenuse being the radius:

$$(R\sin\theta)^2+H^2=R^2.$$

$$\text{So}\quad H^2=R^2*\left(1-(\sin\theta)^2\right)$$

$$H^2=R^2(\cos\theta)^2$$

$$H=R\cos\theta.$$

We now apply the Axiom of Snefru in the canonical right triangle

$$\frac{H}{R}=\frac{2R\sin\theta}{2b}.$$

So $2b=\frac{2R\sin\theta}{\cos\theta}$.

Similarly, we find the length of the longer side of the inscribed right triangle in Figure 7:

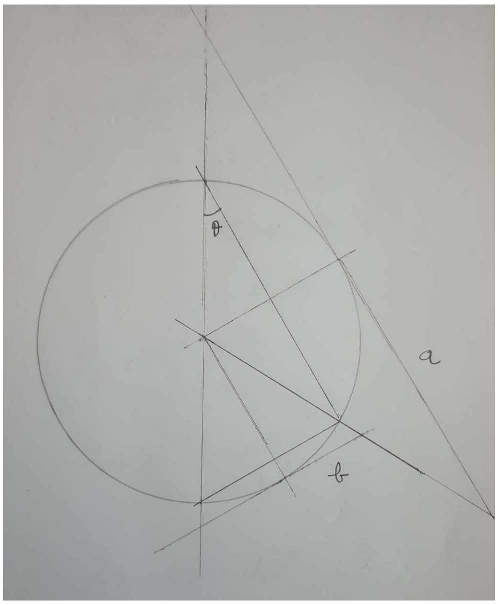

Figure 7. The "any" case.

$$\text{hypotenuse} * \cos\theta = 2R\cos\theta .$$

We apply the Axiom of Ur in the right triangle with the hypotenuse being the radius:

$$(R\cos\theta)^2 + h^2 = R^2 .$$

$$\text{So} \quad h^2 = R^2 * \left(1 - (\cos\theta)^2\right)$$

$$h^2 = R^2 (\sin\theta)^2$$

$$h = R\sin\theta .$$

We now apply the Axiom of Snefru in the canonical right triangle

$$\frac{h}{R} = \frac{2R\cos\theta}{2a} .$$

So $2a = \dfrac{2R\cos\theta}{\sin\theta}$.

Now we can calculate the product $2a * 2b$:

$$2a * 2b = 4R^2 \quad (4)$$

What is equivalent to

$$a * b = R^2 \quad (3)$$

5. Equanimous Inverses

$$\sqrt{x^2+1} + x \quad (5)$$

We consider the function associated with the variable x

$$x \to \sqrt{x^2+1} + x \quad x \in \mathbb{R} .$$

We calculate its inverse:

$$\frac{1}{\sqrt{x^2+1}+x}=\frac{\sqrt{x^2+1}-x}{x^2+1-x^2}=\sqrt{x^2+1}-x\,.$$

Result found by applying the formula

$$(a+b)*(a-b)=a^2-b^2 \quad a \text{ et } b\in\mathbb{R}^2\,.$$

We now consider the difference between the starting function and its inverse:

$$\sqrt{x^2+1}+x-\left(\sqrt{x^2+1}-x\right)=2x\,.$$

We find Equanimous Inverses when this difference is an integer, in other words for $x=\frac{n}{2}\quad n\in\mathbb{N}$.

We apply this change of variable to the starting function:

$$\sqrt{\frac{n^2}{4}+1}+\frac{n}{2}=\frac{\sqrt{n^2+4}+n}{2} \tag{6}$$

Indeed for $n=1$ we find the Golden ratio

$$n=1\quad \frac{\sqrt{5}+1}{2}=1.6180339887498948482045868343656\cdots$$

Approximate result expressed in base ten, that is to say in. A like all results on this page.

However, it turns out that its inverse:

$$\frac{2}{\sqrt{5}+1}=0.6180339887498948482045868343656\cdots$$

What makes the Golden ratio the first Equanimous Inverse apart from 1. We notice here, concerning the Golden ratio, that its inverse is also the Golden ratio minus 1. It also has the particular characteristic that its square is equal to the Golden ratio plus 1.

$$\left(\frac{\sqrt{5}+1}{2}\right)^2=\frac{5+1+2\sqrt{5}}{4}=\frac{3+\sqrt{5}}{2}=\frac{\sqrt{5}+1}{2}+1\,.$$

Let us return to our Equanimous Inverses, and consider now the previous function for the successive values of n:

$$n=2\quad \frac{\sqrt{8}+2}{2}=\sqrt{2}+1=2.4142135623730950488016887242097\cdots$$

$$\text{Or}\quad \frac{1}{\sqrt{2}+1}=0.4142135623730950488016887242097\cdots$$

$$n=3\quad \frac{\sqrt{13}+3}{2}=3.3027756377319946465596106337352\cdots$$

$$\text{Or}\quad \frac{2}{\sqrt{13}+3}=0.3027756377319946465596106337352\cdots$$

$$n=4 \quad \frac{\sqrt{20}+4}{2}=\sqrt{5}+2=4.2360679774997896964091736687313\cdots$$

$$\text{Or} \quad \frac{1}{\sqrt{5}+2}=0.2360679774997896964091736687313\cdots$$

$$n=5 \quad \frac{\sqrt{29}+5}{2}=5.1925824035672520156253552457701\cdots$$

$$\text{Or} \quad \frac{2}{\sqrt{29}+5}=0.1925824035672520156253552457701\cdots$$

$$n=6 \quad \frac{\sqrt{40}+6}{2}=\sqrt{10}+3=6.1622776601683793319988935444327\cdots$$

$$\text{Or} \quad \frac{1}{\sqrt{10}+3}=0.1622776601683793319988935444327\cdots$$

$$n=7 \quad \frac{\sqrt{53}+7}{2}=7.1400549446402591355486512457635\cdots$$

$$\text{Or} \quad \frac{2}{\sqrt{53}+7}=0.1400549446402591355486512457635\cdots$$

And so on for all the values of n belonging to $\mathbb{N}$.

Note that all the numbers on this page are written base ten, but the decimal parts are the same regardless of the base.

6. Conclusion

According to Professor Reygan Dionisio we already know a few theorems on the circles (Figure 8), but none will be The Theorem of The Circle.

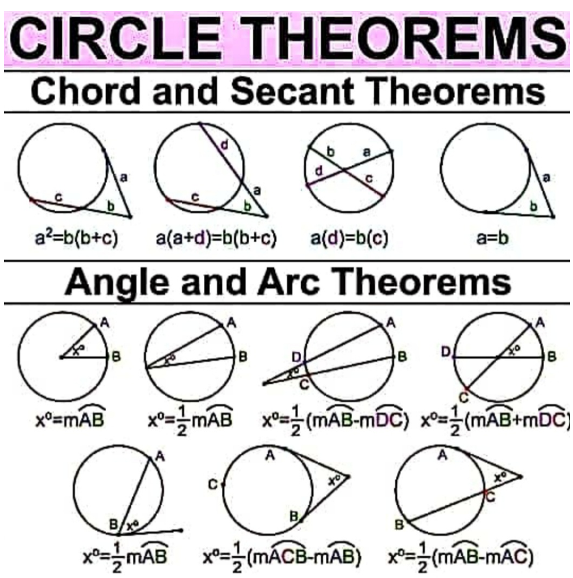

Figure 8. The theorems on circles[2].

[2]Professor Reygan Dionisio (2024) on his Twitter/X account @ZahlenRMD.

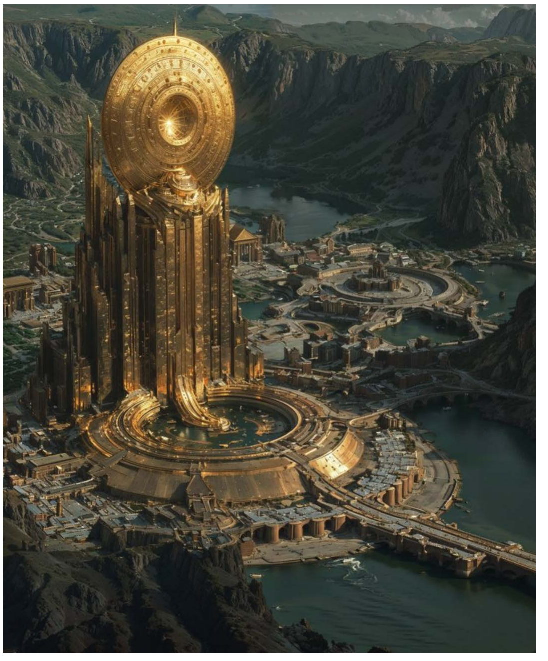

Figure 9. "The Circle" theorem sanctuary[3].

The reciprocal of the Circle Theorem is fundamental to a new mathematical way to draw a Circle (See Figure 9).

The Circle Theorem also states that $2a * 2b = 4R^2$. (4)

Apollon's Theorem could inspire such property with the sphere, maybe with the help of Computational Cartesian Analytic Geometry. Nevertheless no archeologist nor Egyptologist was able until now to figure out properly how is engineered the translation box at the end of the steering rod, never to be confused with the drawbar.

Test IA or YA: "Trouve de quoi être $a * b = R^2$".

Conflicts of Interest

The author declares no conflicts of interest.

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